

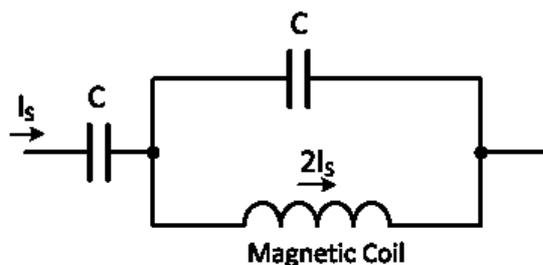
# Section 1: Why High Frequency Magnetic Field Generation is Difficult

Generate high frequency and high strength magnetic field is difficult. As the frequency is increase, so is the electromagnet coil impedance. To generate high magnetic field, high current is needed. Because of high impedance, high voltage and high current driver is needed. Such drivers or amplifier are not readily available. For example, to drive 8A into a 500uH magnetic coil at 250kHz, the amplifier driver must be rated for 6280V and 50kW!

Using classic [series resonant to generate magnetic field](#) can partially solve the above problem by lowering the impedance and therefore lower the amplifier driver voltage requirement. However the maximum current is limited by the magnetic field generator amplifier driver output current capability. The new resonant circuit discussed below will further increase the coil current by 2X.

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To generate high-frequency and high magnetic field, a new resonant circuit is discussed in this Application Note. This App Note uses a newly discovered resonant circuit to magnify the magnetic coil current by a factor of 2. Thus the generated magnetic field is doubled while operating at high-frequency. The new resonant circuit is called current-amplified resonant.

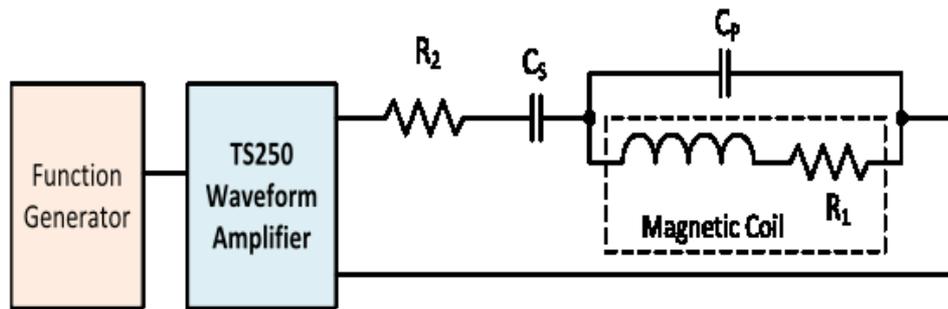


To generate high magnetic field, a waveform amplifier is needed to drive the resonant circuit as shown in Figure 2. The function generator produces a sinewave. The sinewave is amplified by a driver amplifier such as the TS200 or the TS250. It can amplify the function generator voltage or current or both. The waveform amplifier is driving a large current into the current-amplified resonant circuit. The magnetic coil current is twice the amplifier output current!

For example, the TS250-1 maximum (peak) output current is 4.4A, but the magnetic coil current is 8.8A (twice)! In conclusion, the newly discovered current-amplified resonant circuit is acting like a current multiplier. It amplifies the source current by a factor of 2 and therefore it increases the generated magnetic field by a factor of two as well. The combination of waveform amplifier and resonant circuit is ideal for high frequency magnetic field generator.

The series resistance  $R$  is optional. It is for better impedance matching. See Maximizing Amplifier Output Current application note for details how to choose the optimal resistance. In general the series resistor can be omitted.

Figure 1 shows the new resonant circuit. It is consisted of two equal value capacitors, labeled  $C$ , and a magnetic coil. At resonant the magnetic coil current is twice as much as the input source current from any amplifier. Section 2 below discussed the mathematical theory behind the current amplification.



Current-Amplified Resonant Figure 4 is showing the new current-amplified resonant tank. This new resonant circuit offers low reactance at resonance and it amplifies the input source current for the high-frequency electromagnet. In effect the new resonant circuit is a powerful high frequency magnetic field generator. As demonstrated in the next section, the current magnification is two times the amplifier/driver output current. Furthermore the new current-amplified resonant tank is an impedance transformer. At resonance it transforms the magnetic coil parasitic resistance,  $R$ , by a factor of four.

## Newly Discovered Current-Amplified Resonant Circuit

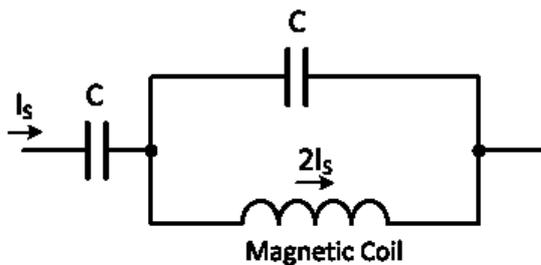


Figure 1. New resonant circuit doubles the magnetic coil current and magnetic field.

## Amplified Resonant

The classic series and parallel LC resonant circuit have been used in numerous applications such as oscillators and filters. LC resonant tank can also be used for magnetic field generator. A new resonant circuit is studied in this App Note for the purpose of generating high frequency magnetic field. The most important feature of this new LC resonant tank is it amplifies the magnetic coil current by 2X while generating high-frequency magnetic field.

### Classic LC Resonant Circuit

Figure 3 shows the series and parallel resonant circuits. The series LC resonant tank features low impedance at resonance. The coil impedance is cancelling the capacitor impedance. Thus achieving low impedance enables high current through the LCR circuit. Up to now, the most practical and efficient way to drive high current through magnetic coil is using series resonant circuit. Thus series resonant technique is often used for high frequency magnetic field generator.

The parallel resonant circuit on the other hand, its impedance is maximum at resonance. At resonant the current is resonating between the coil and capacitor. The current passing through the magnetic coil is very high while the source current is very low - hence high impedance. Thus the parallel resonant circuit amplifies the source current at resonance. However, due to high impedance at resonance, the coil current is generally small even with the current amplification

effect.

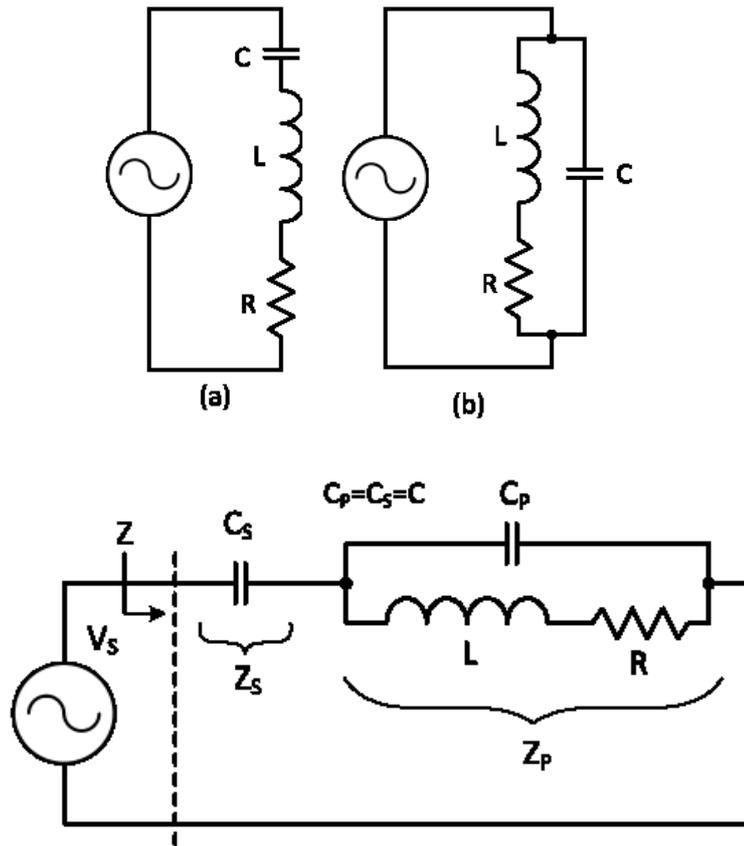


Figure 4. Novel current-amplified resonant circuit showing each impedance.

To calculate the resonant frequency, first we need to define the resonant conditions. The requirements for resonant is such that at a given frequency the imaginary (reactive) portion of the input impedance,  $Z$ , is equal zero. The impedance is purely resistive at resonance.

$$\text{Im}g(Z) = 0 \quad \text{Eq. 1}$$

$$\text{Im}g(Z_P) + \text{Im}g(Z_S) = 0 \quad \text{Eq. 2}$$

In Figure 4 the electromagnetic coil resistance is usually small value and parasitic to the coil. The small resistance has no measurable effect on the tank resonant frequency. To simplify the resonant frequency calculation, the small resistance is excluded. In general magnetic coil with small resistance is better for high strength magnetic field generation. The two capacitors forming the resonant circuit are the same value ( $C_S = C_P = C$ ). Looking at  $Z_P$  only, it is a classic parallel resonant. Using equation A3 from the Appendix section below, the imaginary portion of  $Z_P$  is given in Equation-3. In addition,  $Z_S$  is just a capacitor. For a simple capacitor, the imaginary part of  $Z_S$  is just its reactance giving in Equation-4.

$$\text{Im}g(Z_P) = \frac{1}{\omega C - \frac{1}{\omega L}} \quad \text{Eq. 3}$$

$$\text{Im}g(Z_S) = \frac{1}{\omega C} \quad \text{Eq. 4}$$

Now substitute Equation-3 and Equation-4 into the previous Equation-2 (resonant condition), it becomes Equation-5.

$$\frac{1}{\omega C - \frac{1}{\omega L}} + \frac{1}{\omega C} = 0 \quad \text{Eq. 5}$$

$$\frac{1}{\omega C - \frac{1}{\omega L}} = -\frac{1}{\omega C}$$

$$\omega C - \frac{1}{\omega L} = -\omega C$$

$$2\omega C = \frac{1}{\omega L}$$

Next is solving for frequency  $\omega$ . The result is showing on Equation-6 which is the current-amplified resonant frequency  $\omega_0$ .

$$\omega_0 = \frac{1}{\sqrt{2LC}} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{LC}} \quad \text{Eq. 6}$$

$$f_0 = \frac{1}{\sqrt{2}} \left( \frac{1}{2\pi\sqrt{LC}} \right) \quad \text{Eq. 7}$$

$$C = \frac{1}{2L(2\pi f_0)^2} \quad \text{Eq. 8}$$

Remember  $\omega_0 = 1/\sqrt{LC}$  is the classic (series or parallel) resonant frequency of Figure 3. Rearranging Equation-6 to Equation-7, the new current-amplified resonant frequency is related to the classic series or parallel resonant frequency by a factor of  $1/\sqrt{2}$  times. The reason for the new current-multiply resonant frequency is lower, because the new resonant has two capacitors. Further rearranging Equation-7 to calculated the capacitance for a given resonant frequency and coil inductance as shown in Equation-8

As mentioned above, the new current-amplified resonant is differed from the classic resonant is that it amplifies the electromagnetic coil current 2 times at resonance. That is to say, the resonant circuit doubles the source amplifier driver current. Hence it doubles the magnetic field generated. To understand how the solenoid magnetic coil current is amplified, Kirchhoff's voltage law (KVL) is used as shown in Figure 5. Kirchhoff's voltage law (KVL) is applied in the parallel resonant loop (CP, L, and R). To simplify the calculation, the small coil resistance R is ignored again. Also CP and CS are equal value for this new resonant.

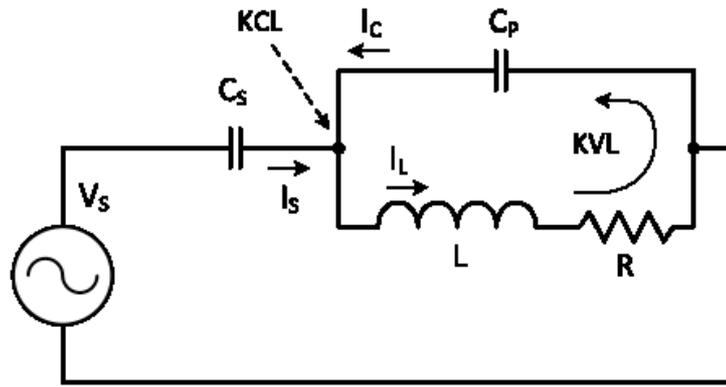


Figure 5.KVL and KCL are used to calculate the electromagnet coil (inductor) current.

$$I_L(j\omega L) + I_C \frac{-j}{\omega C} = 0$$

$$I_L(\omega L) = \frac{I_C}{\omega C}$$

$$I_L = \frac{I_C}{\omega C} \frac{1}{\omega L} = \frac{I_C}{\omega^2 LC} \quad \text{Eq. 9}$$

Remember from Equation-6, the resonance frequency is  $\omega=1/\sqrt{2LC}$ . Now substitute Equation-6 into Equation-9 and simplify.

$$I_L = I_C \frac{1}{\left(\frac{1}{\sqrt{2LC}}\right)^2 LC} = 2I_C \quad \text{Eq. 10}$$

From Equation-10 and Figure 5 the magnetic coil current is two times the capacitor CP or CS current. From the illustration in Figure 5, the two capacitor currents are flow into the coil at the same time. Thus the new resonant tank is achieving current magnification.

To further rearrange Equation-10 in term of source current, Kirchhoff's current law (KCL) is used. Recall the sum of all current into a node is zero as stated by KCL. Equation-11 is the KCL at the node where all three currents (IS, IC, IL) are met. From Equation-10 IC is one-half of IL and substitute IC from Equation-10 into equation-11 results:

$$I_L = I_S + I_C \quad \text{Eq. 11}$$

$$I_L = I_S + \frac{1}{2} I_L$$

$$I_L = 2I_S \quad \text{Eq. 12}$$

In conclusion from Equation-12, the magnetic coil current (IL) is amplified and is twice the signal source amplifier driver current IS. At resonant the magnetic coil current is two times the

maximum available current from any given signal source! Therefore the magnetic field density is also twice as much! This is a very important feature of the new resonant circuit for scientific experiments that require high-strength and high-frequency magnetic field generation. To visually understand the current-doubling effect, the current flow is shown Figure 6. The inductor current is the sum of the two capacitor currents.

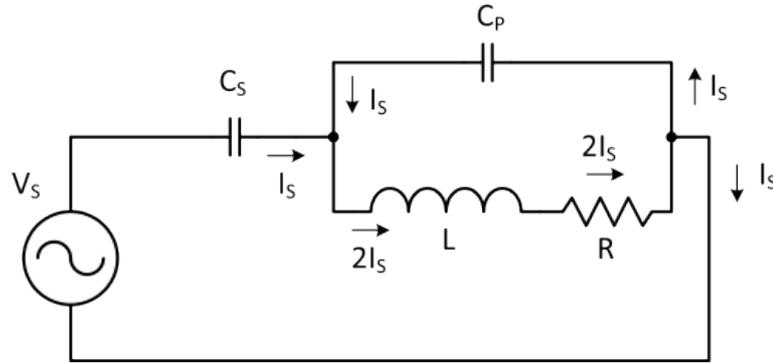


Figure 6. Visual depiction of the magnetic coil current is two times the source current.

To understand the impedance of the resonant tank circuit, consider the reactive (imaginary part) impedance  $Z$  is zero as previously mentioned in Equation-1 and Figure 4. Therefore the new current-amplified resonant impedance is real (resistive) at resonance. Because  $Z_S$  is just a capacitor and reactive only, the real part (resistive) of  $Z$  is coming from  $Z_P$ . From the Appendix Equation-A2, the real part of  $Z$  is expressed in Equation-13. Recall  $\omega = 1/\sqrt{2LC}$  at resonant, and substitute it into Equation-13 then simplify yields Equation-14.

$$R_{EQ} = \frac{\frac{R}{(\omega C)^2}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)} \quad \text{Eq. 13}$$

$$R_{EQ} = \frac{2\frac{RL}{C}}{R^2 + \frac{L}{2C}} \quad \text{Eq. 14}$$

Because the  $L/(2C)$  term is much bigger than the  $R^2$  term, The  $R^2$  term is ignored. Equation-14 is simplified to Equation-15.

$$R_{EQ} \cong 4R \quad \text{Eq. 15}$$

From Equation-15 the magnetic coil parasitic resistance is multiplied by 4. Thus the new current-amplified resonant circuit is an impedance transformer. It transforms the resistance by a factor of 4. Usually electromagnetic coil resistance is small. So even it is multiply by four, the total resistance is still relatively small. When designing coil for high-frequency and high-strength magnetic field generator, low resistance is desirable. Low resistance enables the use of low-voltage, high-current, and high-frequency source amplifier driver such as the TS250.

The main topic of this study is the novel resonant circuit's ability to magnify the input current by a factor of 2 for free. The boosted current is ideal for generating high magnetic field density. From the above equations, the quality factor  $Q$  does not impact the current amplification. To complete the new resonant study, the quality factor  $Q$  is presented here.

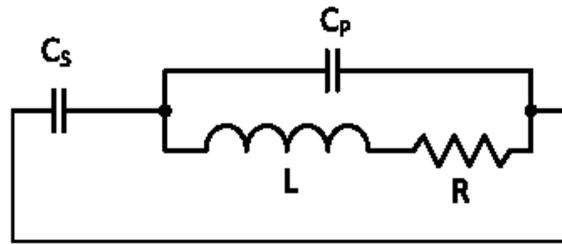


Figure 7. For calculating Q, the current-amplified resonant circuit is simplified to a simple parallel resonant circuit with 2X the capacitance.

Recall the small-signal model of any voltage source, VS, is equivalent to a short circuit. From signal model perspective, the current-amplified resonant circuit in Figure 4 can be redrawn here in Figure 7. As depicted in Figure 7, the two capacitors are connected in parallel with the magnetic coil. This forms a single classic parallel resonant circuit. Remember from classic parallel resonant circuit, the quality factor is given in Equation-16.

$$Q = \frac{\omega L}{R} \text{ Eq. 16}$$

In conclusion the new current-amplified resonant circuit quality factor, Q, is same as the classic parallel resonant circuit. The only difference is the resonant frequency is lower.

## Section 3

### Magnetic Field Generator Test Results

The current-amplified resonant circuit in Figure 4 can be easily simulated using a SPICE tool. It is a quick way to verify the new resonant circuit mathematical equations discussed in the previous section. In this simulation a 10 $\mu$ H and two 1.26nF capacitors were used to form the resonant. Using Equation-7 the calculated resonance frequency is 1.0026MHz. Upon simulation, the measured resonant is 1.025MHz and it is shown in Figure 9. Figure 8 shows the plot of magnetic coil current vs time. The source current is measured 5A peak and the coil current is measured 10A peak. The current-amplifying effect of the new resonant circuit is confirmed by simulation.

In the SPICE simulation, the coil resistance was set to 125m $\Omega$ . The simulated source voltage was set to 2.5V. The measured source current is 5A. The calculated resonant tank effective resistance is 500m $\Omega$ . As discussed above the resonant circuit transforms the coil resistance by a factor of 4. Simulation confirms Equation-15 (4\*125m $\Omega$  = 500m $\Omega$ ). Using the SPICE simulation tool, it was found the current amplification factor of 2 is maintained even with coil resistance of 10 ohm. SPICE simulation confirms the mathematical model discussed above.

When the source generator current vs frequency is plotted in Figure 10, it reveals two resonances: the new resonant and the classic parallel resonant. The current-amplified resonant frequency is lower than the classic resonant frequency a factor of  $1/\sqrt{2}$  as predicted in Equation-6.

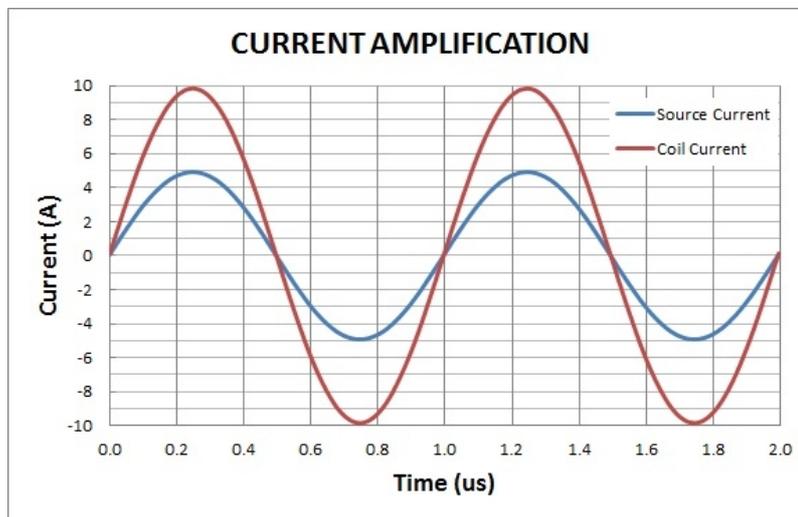


Figure 8. Current-amplifying effect confirmed by the magnetic coil current is twice the source current.

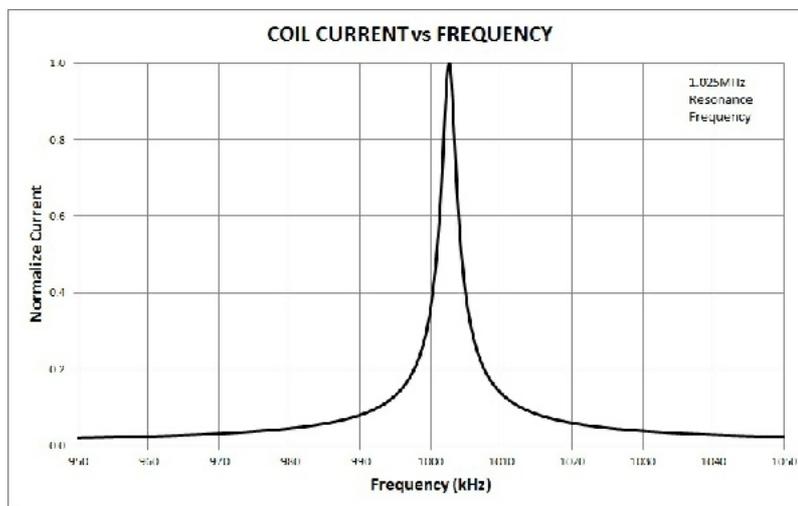


Figure 9. The new resonant frequency is found by the magnetic coil current vs frequency.

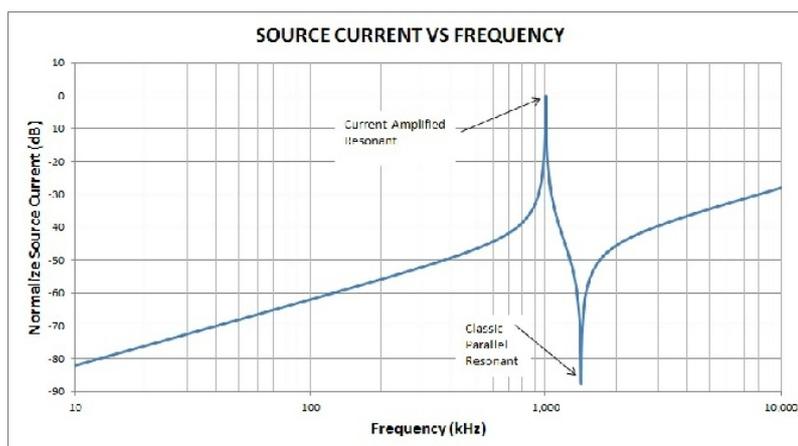


Figure 10. The new current-amplifying resonant actually has two resonance.

An experiment was designed to use a real coil (inductor) to confirm the current -amplifying effect discussed above. The experiment uses the [TS250 high-current Waveform Amplifier](#). In this experiment, the high frequency magnetic coil used was a 100uH high-current inductor and the two capacitors are 10nF each. Because of high current and high frequency, the two capacitors must be rated for high voltage. They are rated 1kV in this experiment.

Figure 11 shows the experiment setup. To validate the experiment, source current and coil current must be measured. Two  $1\Omega$  resistors,  $R_1$  and  $R_2$ , was used to monitor the currents. Because the voltage drops across  $R_1$  and  $R_2$  are proportional to the current, current is measured by measuring the voltages. For example,  $1V$  across the resistor is equal to  $1A$ .

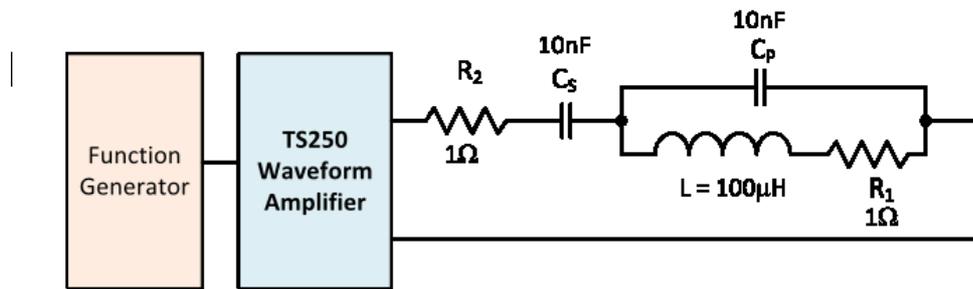


Figure 11. TS250 Waveform Amplifier used to generate high frequency magnetic field.

The first step is to set the function generator and the TS250 amplifier driver to the calculated resonant frequency ( $112.5kHz$ ) with a  $10V_{pp}$  sinewave. Next find the resonant frequency by slowly adjust the frequency until the coil reaches maximum current. The resonant frequency was found to be  $116.5kHz$ . Now slowly increase the magnetic coil current by increase the TS250 peak-to-peak voltage. When the Waveform Amplifier voltage reaches  $30V_{pp}$ , the input current is about  $3A_{pp}$  as shown in Figure 12. The magnetic coil current was measured  $6A_{pp}$  (Figure 13). This verified inductor coil current is amplified by the resonant and it doubles the input current from the Waveform Amp.

Using Equation-15 the calculated load impedance (resonant circuit) is  $4\Omega$  (exclude  $R_2$ ). Because a  $30V_{pp}$  voltage produces  $3A_{pp}$ , the experimental resonant resistance is  $10\Omega$ . Furthermore the specified inductor DC resistance is only  $25m\Omega$ . The mismatch between the experimental resistance ( $10\text{ ohm}$ ) and theoretical resistance ( $\sim 4\text{ ohm}$ ) is probably due to the inductor's AC resistance at  $116kHz$  is much higher. The higher AC resistance is likely cause by the skin effect. Furthermore the capacitors also add parasitic AC resistance at high frequency. In summary, for high-current and high magnetic field generator, choose a coil with the least DC and AC resistance.

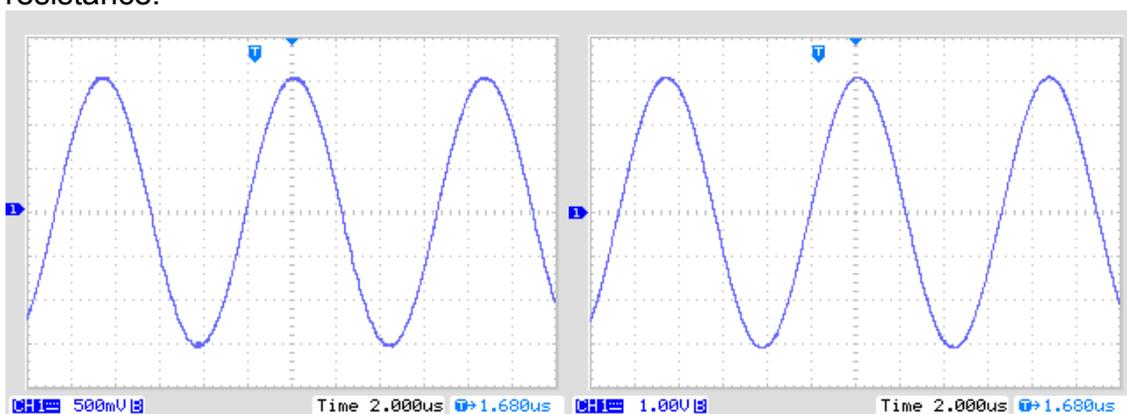


Figure 12.  $3A_{pp}$  input source current is measured across  $R_2$ .  $1V = 1A$ .  
 Figure 13.  $6A_{pp}$  magnetic coil current is measured across  $R_1$ .  $1V = 1A$ .

### Amplified Resonant Conclusion

The newly discovered current-amplified resonant circuit discussed here is a powerful way to produce high frequency and high density magnetic field. This magnetic field generation technique is ideal for scientific and R & D experiments. The current-amplifying effect is

presented in the mathematical model discussed above. SPICE simulation was first used to confirm it and then lab experiments verify the magnetic coil current is in fact doubled by the current-amplifying resonant circuit.

## Parallel Resonant Impedance Derivation

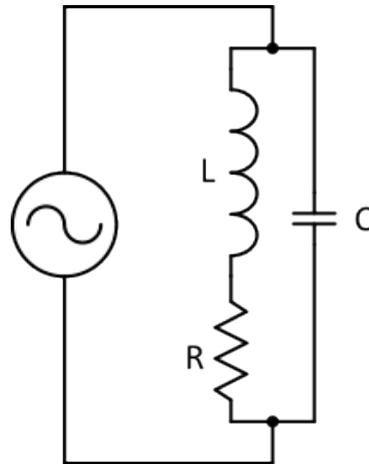


Figure A. Classic parallel resonant circuit consisted of RLC for calculating impedance and reactance.

$$Z = \frac{1}{z_L} + \frac{1}{z_C} = \frac{z_L z_C}{z_L + z_C}$$

$$Z_L = R + j\omega L$$

$$Z_C = \frac{-j}{\omega C}$$

$$Z = \frac{(R + j\omega L) \left(\frac{-j}{\omega C}\right)}{R + j\omega L - \frac{j}{\omega C}}$$

$$Z = \frac{\frac{L}{C} - \frac{jR}{\omega C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$Z = \frac{\frac{L}{C} - \frac{jR}{\omega C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \left( \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R - j\left(\omega L - \frac{1}{\omega C}\right)} \right)$$

$$Z = \frac{\frac{RL}{C} - R\left(\frac{L}{C} - \frac{1}{(\omega C)^2}\right) - j\left(\frac{R^2}{\omega C} + \frac{\omega L^2}{C} - \frac{L}{\omega C^2}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$Z = \frac{R}{(\omega C)^2 - j\left(\frac{R^2}{\omega C} + \frac{\omega L^2}{C} - \frac{L}{\omega C^2}\right)} \quad \text{EQ. A1}$$

$$Re(Z) = \frac{\frac{R}{(\omega C)^2}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \text{EQ. A2}$$

$$Img(Z)|_{R=0} = \frac{\left(\frac{\omega L^2}{C} - \frac{L}{\omega C^2}\right)}{\left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{\frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right)}{\left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{\frac{L}{C}}{\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{\left(\omega C - \frac{1}{\omega L}\right)} \quad \text{EQ. A3}$$

## Coil Amplifier Driver Selection Guide

As shown in Figure 2, high frequency magnetic field is generated by a waveform amplifier driving the resonant coil. The TS200 Modulated Power Supply or the TS250 series of Waveform Amplifier are ideal for driving high current through magnetic coils. Select an amplifier driver (Table 1) based on the equivalent coil resistance at resonant using Equation-15. The coil resistance is specified at resonant frequency. Generally coil resistance is higher at higher frequency due to the Skin-Effect.

Waveform Amplifier	Voltage Range (V)	Coil Resistance (Ohm)	Resonant Resistance (Ohm)	Peak Amplifier Current* (A)	Peak Coil Current* (A)
TS250-0	-10 to +10	0 to 0.3	0 to 1.25	-6 to +6.0	-12 to +12
TS250-1	-20 to +20	0.3 to 1.0	1.25 to 4.0	-4.4 to +4.4	-8.8 to +8.8
TS250-2	-30 to +30	1.0 to 2.25	4.0 to 9.0	-3.0 to +3.0	-6.0 to +6.0
TS250-3	-40 to +40	2.25 to 8.8	9.0 to 35	-2.5 to +2.5	-5.0 to +5.0
TS200-0A	-10 to +10	0 to 0.38	0 to 1.5	-4.5 to +4.5	-9.0 to +9.0
TS200-1B	-20 to +20	0.38 to 1.1	1.5 to 4.5	-3.5 to +3.5	-7.0 to +7.0
TS200-5B	-40 to +40	1.1 to 4.75	4.5 to 19	-2.0 to +2.0	-4.0 to +4.0

Source:

<http://www.accelinstruments.com/Applications/WaveformAmp/Magnetic-Field-Generator.html>