Electromagnetic Induction

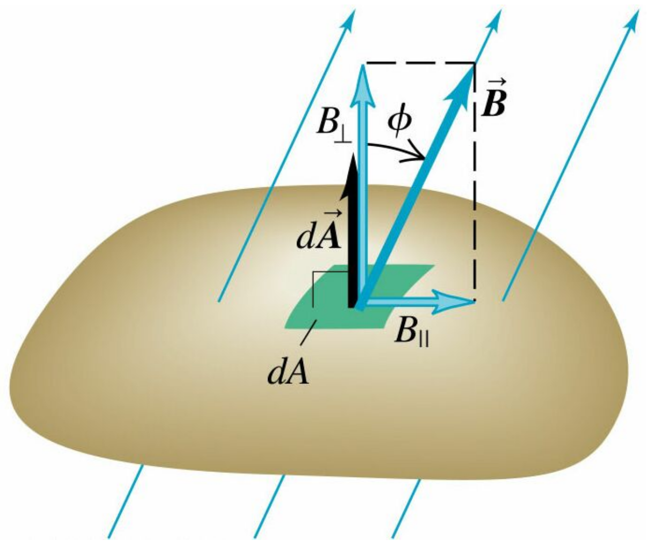
Induced Current

- Past experiments with magnetism have shown the following
  - When a magnet is moved towards or away from a circuit, there is an induced current in the circuit
  - This is still true even if it is the circuit that is moved towards or away from the magnet
  - When both are at rest with respect to each, there is ***no*** induced current
- What is the physical reason behind this phenomena

Magnetic Flux

Just as was the case with electric fields, we can define a magnetic flux

$$d\Phi_B = \vec{B} \cdot d\vec{A}$$



where *dA* is an incremental area

with the total flux being given by

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Note that this integral is ***not*** over a closed surface, for that integral would yield zero for an answer, since there are no sources or sinks for the magnetic as there is with electric fields

Faraday's Law of Induction

It was Michael Faraday who was able to link the induced current with a changing magnetic flux

He stated that

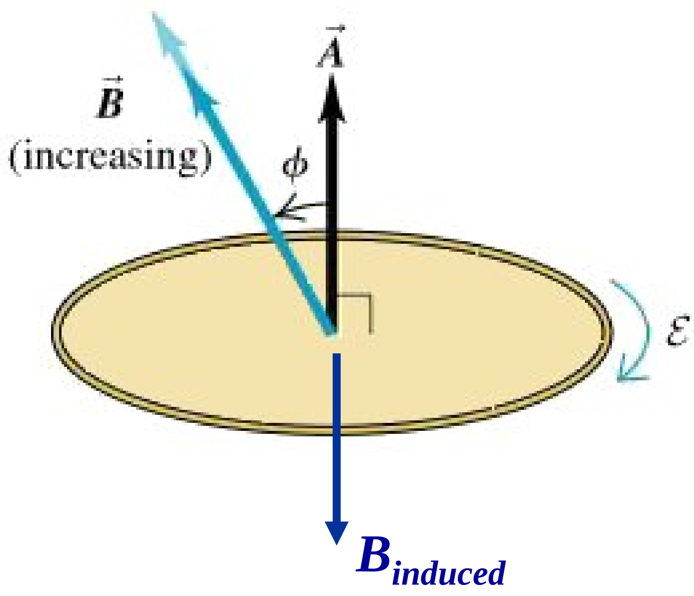
> ***The induced emf in a closed loop equals the negative of the time rate of change of the magnetic flux through the loop***

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The induced emf *opposes* the change that is occurring

Increasing Magnetic Flux

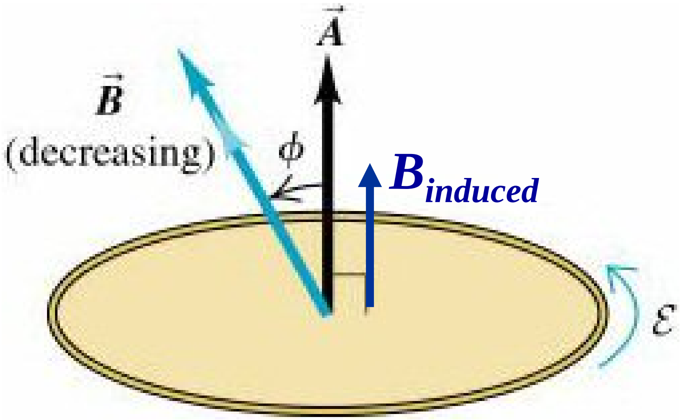
Suppose that we are given a closed circuit through which the magnetic field, *flux*, is increasing, then according to Faraday's Law there will be an induced emf in the loop



The sense of the emf will be so that the induced current will set up a magnetic field that will oppose this increase of external flux

Decreasing Magnetic Flux

Suppose now that the magnetic field is decreasing through the closed loop, Faraday's Law again states that there will be an induced emf in the loop



The sense of the emf will be so that the induced current will set up a magnetic field that will try to keep the magnetic flux at its original value (It never can)

Changing Area

Both of the previous examples were based on a changing external magnetic flux

How do we treat the problem if the magnetic field is constant and it is the area that is changing

Basically the same way, it is the changing flux that will be opposed

If the area is increasing, then the flux will be increasing and if the area is decreasing then the flux will also be decreasing

In either case, the induced emf will be such that the change will be opposed

Lenz's Law

All of the proceeding can be summarized as follows

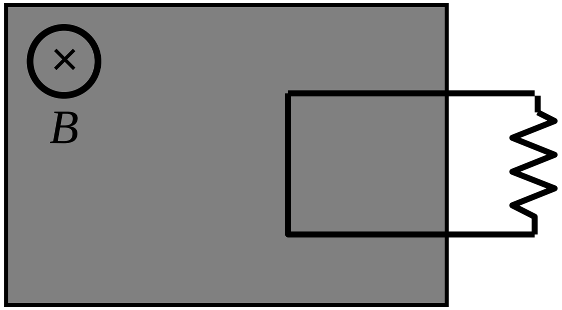
The direction of any magnetic induction effect is such as to oppose the cause of the effect

Remember that the cause of the effect could be either a changing external magnetic field, or a changing area, or *both*

$$d\Phi_B = d\vec{B}\cdot\vec{A} + \vec{B}\cdot d\vec{A}$$

Example

Inside the shaded region, there is a magnetic field into the board.
If the loop is stationary, the Lorentz force predicts:



a) A Clockwise Current b) A Counterclockwise Current c) No Current

The Lorentz force is the combined electric and magnetic forces that could be acting on a charge particle

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Since there is no electric field and the charges within the wire are not moving, the Lorentz force is zero and there is no charge motion

Therefore there is no current

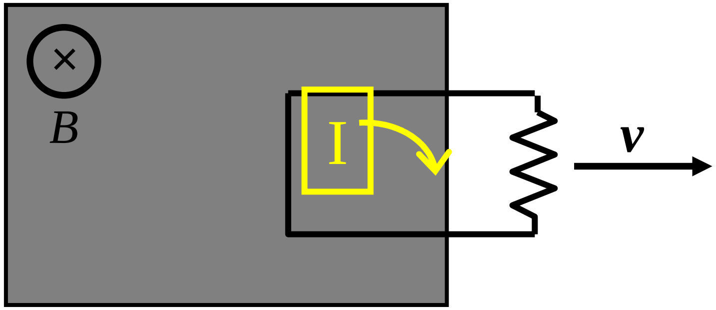
Example - Continued

Now the loop is pulled to the right at a velocity *v*. The Lorentz force will now give rise to:

(a) A Clockwise Current

(b) A Counterclockwise Current

(c) No Current, due to Symmetry

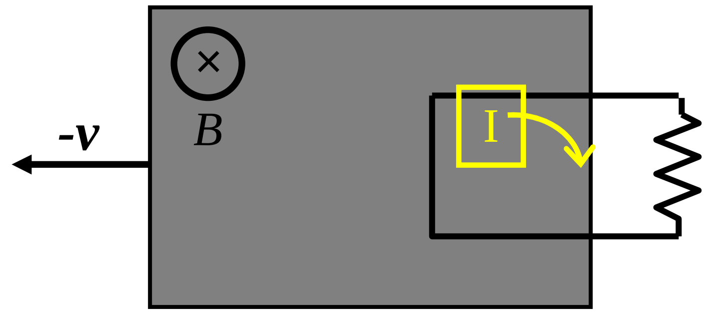


The charges in the wire will now experience an upward magnetic force, $\vec{F} = q\vec{v} \times \vec{B}$, causing a current to flow in the clockwise direction

Another Example

Now move the magnet, not the loop

Here there is *no* Lorentz force $\vec{v} \times \vec{B}$

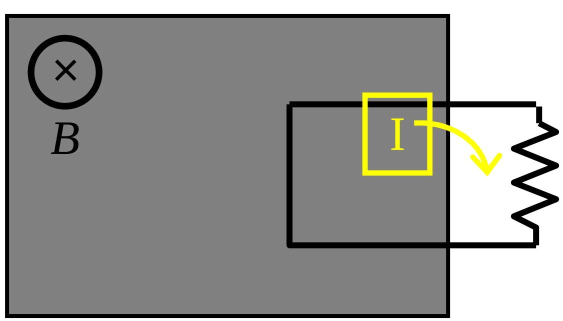


But according to Lenz's Law there will still be an induced current

This "coincidence" bothered Einstein, and eventually led him to the Special Theory of Relativity (all that matters is relative motion).

Now keep everything fixed, but decrease the strength of B

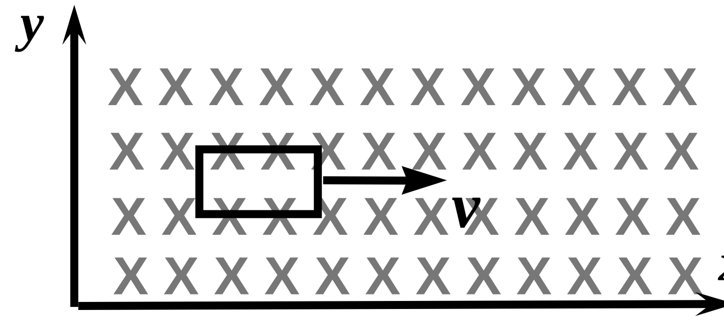
Again by Lenz's Law there will be an induced current because of the decreasing flux



Decreasing *B*

Example

A conducting rectangular loop moves with constant velocity v in the $+x$ direction through a region of constant magnetic field B in the $-z$ direction as shown



What is the direction of the induced current in the loop?

(a) ccw

(b) cw

(c) no induced current

There is a non-zero flux Φ_B passing through the loop since B is perpendicular to the area of the loop

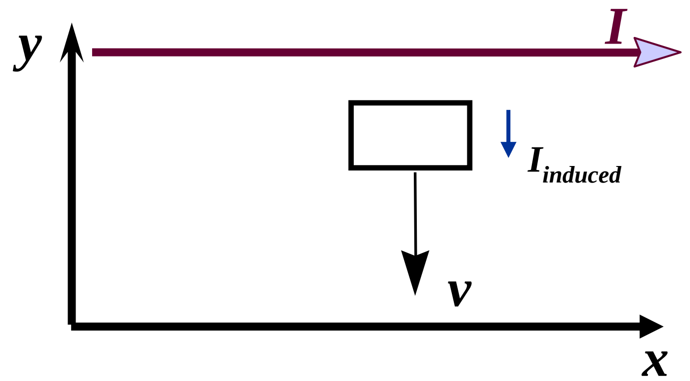
Since the velocity of the loop and the magnetic field are constant, however, the flux through the loop does not change with time

Therefore, there is no emf induced in the loop

No current will flow!!

Example

A conducting rectangular loop moves with constant velocity *v* in the -*y* direction and a constant current *I* flows in the +*x* direction as shown



What is the direction of the induced current in the loop?

(a) ccw

(b) cw

(c) no induced current

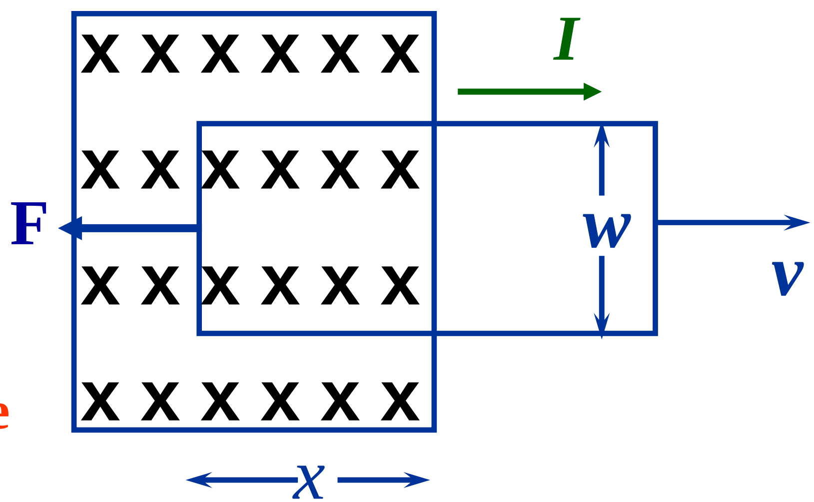
The flux through this loop **does** change in time since the loop is moving from a region of higher magnetic field to a region of lower magnetic field

Therefore, by Lenz's Law, an emf will be induced which will oppose the change in flux

Current is induced in the clockwise direction to restore the flux

(Important) Example

Suppose we pull with velocity v a coil of resistance R through a region of constant magnetic field B.



a) What is the direction of the induced current

By Lenz's Law the induced current will be clockwise in order to try and maintain the original flux through the loop

b) What is the magnitude of the induced current

Magnetic Flux: $\Phi_B = B\,Area = B\,w\,x$

Faraday's Law: $\varepsilon = -\dfrac{d\Phi_B}{dt} = -B\,w\dfrac{dx}{dt} = -B\,w\,v$

Then $I = \dfrac{\varepsilon}{R} = \dfrac{B\,w\,v}{R}$

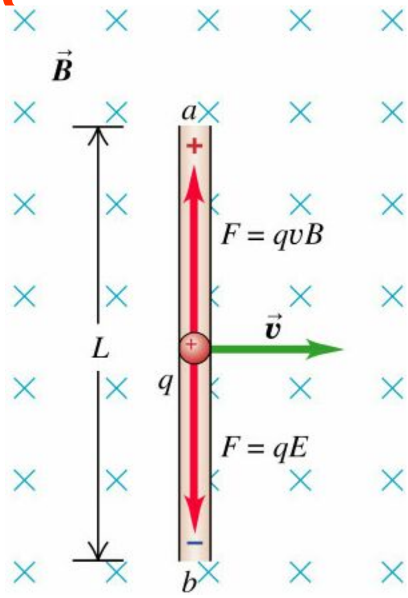
c) What is the direction of the force on the loop?

Using $\vec{F} = I\,\vec{L} \times \vec{B}$, the force is to the left, trying to pull the loop back in!

Motional *EMF*

In previous discussions we had mentioned that a charge moving in a magnetic field experiences a force

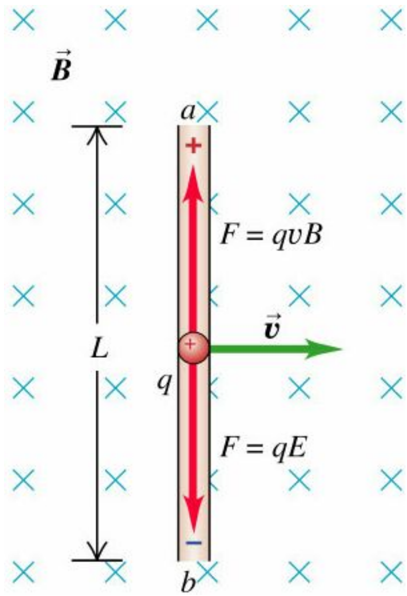
Suppose now that we have a conducting rod moving in a magnetic field as shown



The positive charges will experience a magnetic force upwards, while the negative charges will experience a magnetic force downwards

Charges will continue to move until the magnetic force is balanced by an opposing electric force due to the charges that have already moved

Motional *EMF*



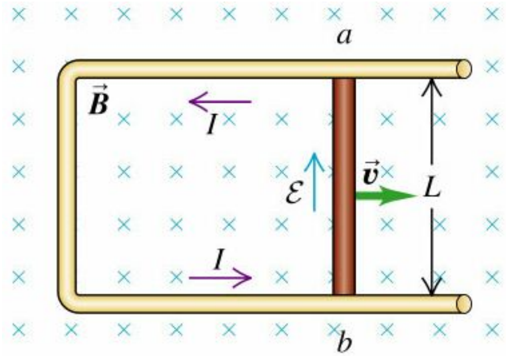
So we then have that $E = vB$

But we also have that $E = V_{ab} / L$

So $V_{ab} = v\,B\,L$

We have an induced potential difference across the ends of the rod

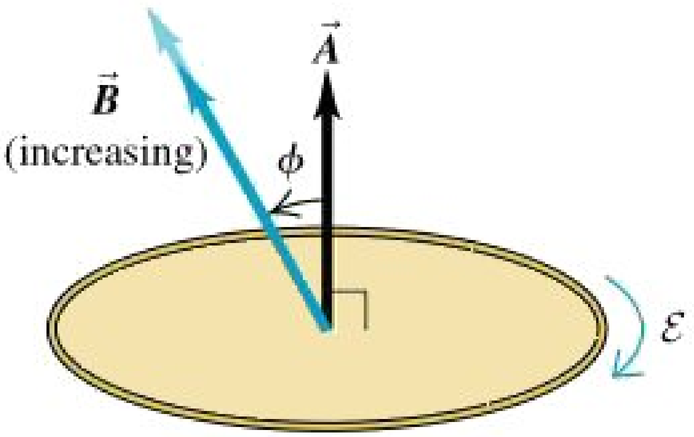
If this rod were part of a circuit, we then would have an induced current



The sense of the induced *emf* can be gotten from Lenz's Law

Induced Electric Fields

We again look at the closed loop through which the magnetic flux is changing



We now know that there is an induced current in the loop

But what is the force that is causing the charges to move in the loop

It can’t be the magnetic field, as the loop is not moving

Induced Electric Fields

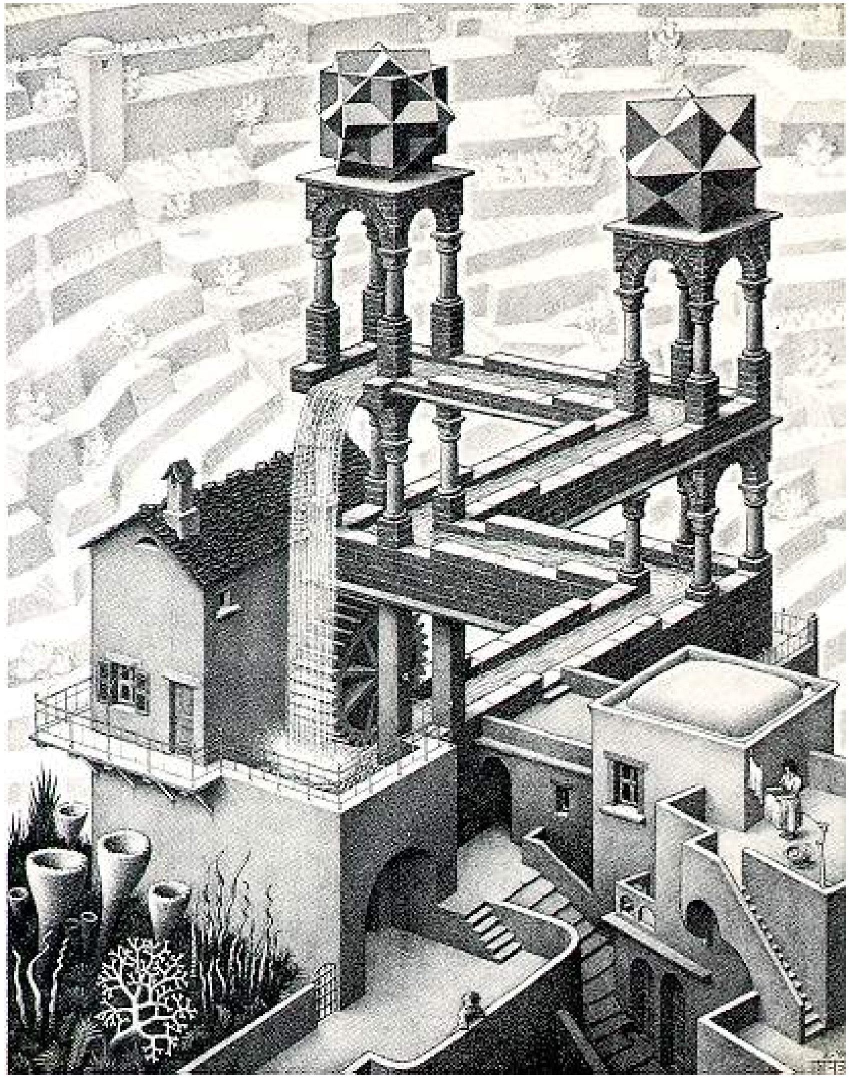
The only other thing that could make the charges move would be an *electric field that is induced* in the conductor

This type of electric field is different from what we have dealt with before

Previously our electric fields were due to charges and these electric fields were conservative

Now we have an electric field that is due to a changing magnetic flux and *this electric field is non-conservative*

Escher's "Depiction" of a Nonconservative *emf*



Induced Electric Fields

Remember that for conservative forces, the work done in going around a complete loop is ***zero***

Here there is a net work done in going around the loop that is given by $q\varepsilon$

But the total work done in going around the loop is also given by

$$q\oint \vec{E} \cdot d\vec{l}$$

Equating these two we then have $\oint \vec{E} \cdot d\vec{l} = \varepsilon$

Induced Electric Fields

But previously we found that the emf was related to the negative of the time rate of change of the magnetic flux

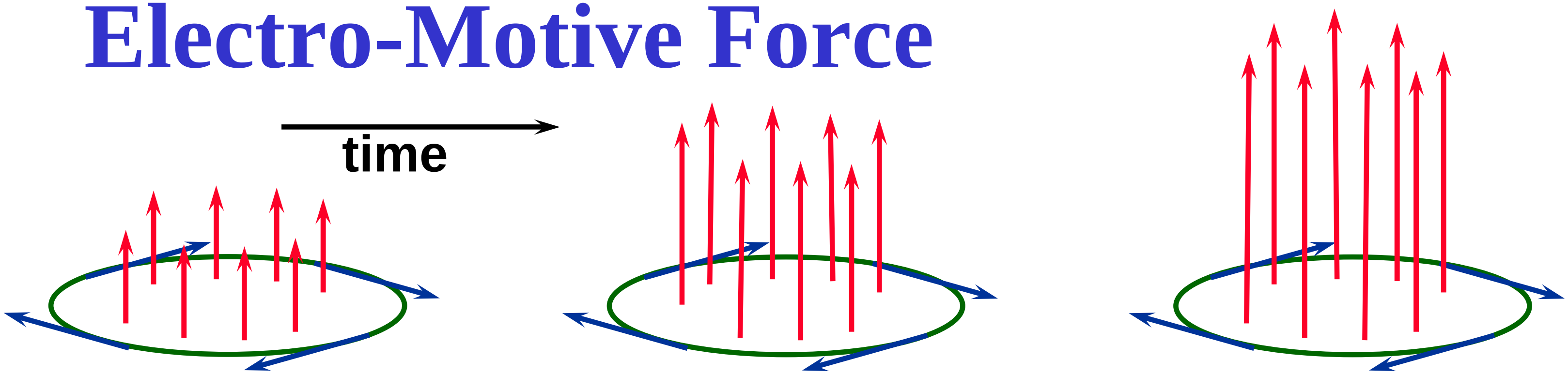
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

So we then have finally that

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

This is just another way of stating Faraday's Law but for stationary paths

Electro-Motive Force



A magnetic field, increasing in time, passes through the loop

An electric field is generated "ringing" the increasing magnetic field

The circulating E-field will drive currents, just like a voltage difference

Loop integral of E-field is the "emf": $\oint \vec{E} \cdot d\vec{l} = \varepsilon$

The loop does not have to be a wire - *the emf exists even in vacuum*!

When we put a wire there, the electrons respond to the emf, producing a current

Displacement Current

We have used Ampere's Law to calculate the magnetic field due to currents

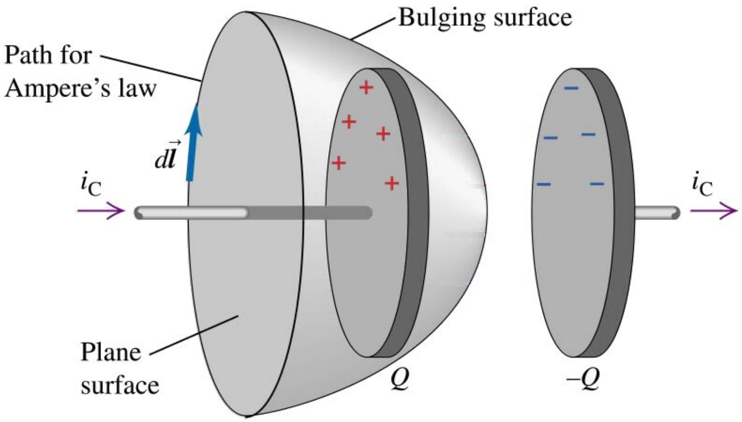
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

where $I_{enclosed}$ is the current that cuts through the area bounded by the integration path

But in this formulation, Ampere's Law is *incomplete*

Displacement Current

Suppose we have a parallel plate capacitor being charged by a current I_c



We apply Ampere's Law to path that is shown

For this path, the integral is just

$$\mu_0 I_{enclosed}$$

For the plane surface which is bounded by the path, this is just I_c

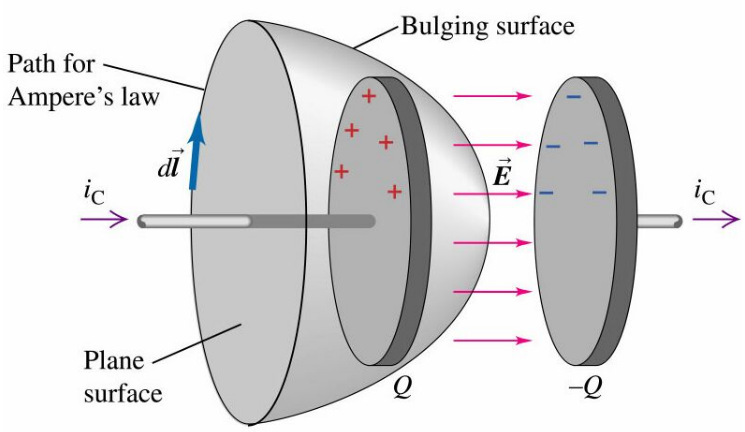
But for the bulging surface which is also bounded by the integration path, $I_{enclosed}$ is zero

So how do we reconcile these two results?

Displacement Current

But we know we have a magnetic field

Since there is charge on the plates of the capacitor, there is an electric field in the region between the plates



The charge on the capacitor is related to the electric field by

$$q = \varepsilon E A = \varepsilon \Phi_E$$

We can define a "current" in this region, ***the displacement current***, by

$$I_{Displacement} = \frac{dq}{dt} = \varepsilon \frac{d\Phi_E}{dt}$$

Displacement Current

We can now rewrite Ampere's Law including this displacement current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Maxwell's Equations

We now gather all of the governing equations together

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad (\text{Gauss's law for } \vec{E})$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad (\text{Gauss's law for } \vec{B})$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad (\text{Faraday's law})$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \qquad (\text{Ampere's law})$$

Collectively these are known as Maxwell's Equations

Maxwell's Equations

These four equations describe all of classical electric and magnetic phenomena

Faraday's Law links a changing magnetic field with an induced electric field

Ampere's Law links a changing electric field with an induced magnetic field

Further manipulation of Faraday's and Ampere's Laws eventually yield a second order differential equation which is the wave equation with a prediction for the wave speed of

$$\boldsymbol{c} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \ \boldsymbol{m} / \sec = \textbf{Speed of Light}$$